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Is the Cosmological Singularity Thermodynamically Possible?

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The four broad approaches that have been suggested heretofore to eliminate the initial singularity from cosmology are briefly reviewed. None is satisfactory, basically because one does not know enough about the microphysics involved in the process. Thermodynamics has often been used in such dilemmas, and it is proposed to answer the question of whether there was a Friedmann-like singularity in the universe by exploiting the bound on specific entropy that has been established for finite system. It is made applicable to the universe by considering only a causally connected spacelike region within the particle horizon of a given observer. It is found that the specific entropy of radiation in such a region can exceed the bound if the observer is too early in the universe. Faith in the bound leads to the conclusion that the Friedmann models cannot be extrapolated back to nearer than a few Planck–Wheeler times from the singularity. The Friedmann initial singularity thus appears to be thermodynamically unacceptable.

1. INTRODUCTION

With the discovery of the expansion of the universe (Hubble, 1929) Friedmann's cosmological models (Friedmann, 1922, 1924) were shown to be the right framework for description of the recent universe. Can they be extrapolated to the very beginnings of the universe? The cosmic microwave background certifies that the models are good back to redshift $z = 10^3$. The success of the standard Friedmann model in quantitatively explaining the cosmological helium abundance pushes the region of validity back to maybe $z = 10^9$. The last decade has witnessed great strides in pushing the earliest redshift to redshift 10^{32}—the Planck era.

At the end of this cosmic tunnel looms the initial cosmological singularity. It is the consensus in gravitational theory and cosmology that accepting the singularity as a real feature of the universe brings with it thorny issues:

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lack of predictability, etc. Most cosmologists and gravity theorists would argue that the singularity will not show up in a correct treatment of the expansion. In what follows I would like to briefly review, in qualitative terms and with no pretense to completeness, the four roads that have been tried in the past to do away with the singularity. It will become clear that no road guarantees excision of the singularity. Then I would like to describe a little argument that suggests that the singularity is thermodynamically forbidden. This is based on the bound on specific entropy whose existence I suggested a few years ago. If one takes the argument seriously, one learns that whatever the mechanism responsible, the universe is loath to begin from a singularity.

2. THE SINGULARITY

The cosmological singularity is a common feature of a large class of cosmological models. In the Friedmann models it describes a state of infinite temperature, density, and arbitrarily large curvature of spacetime at the beginning of the expansion. Similar singularities occur in other cosmological models. For example, in most models based on the inflationary paradigm (Guth, 1981; Linde 1982) the universe begins in a Friedmannian phase before embarking on the de Sitter inflationary phase. The ubiquity of the initial singularity is, to some extent, guaranteed by the "singularity theorems" (Hawking, 1965, 1966; Geroch, 1966; Penrose and Hawking, 1970; Hawking and Ellis, 1973). They show that any cosmological model based on Einstein's equations and satisfying several loose conditions must have a singularity. One of these conditions is the strong energy condition

$$(T^{\mu\nu} - 1/2g^{\mu\nu}T)u_\mu u_\nu > 0 \tag{1}$$

on the stress energy tensor $T^{\mu\nu}$ of matter and its trace T; it is supposed to hold for every timelike vector u_μ. Although the singularity theorems do not specify the nature of the singularity, there is much evidence that in models closely resembling Friedmann's in the geometry of space and in the contents of the universe, an infinite density and temperature singularity is present.

3. EXOTIC PROPERTIES OF MATTER

The singularity theorems are often called upon as evidence that quantum cosmology is needed to resolve the singularity dilemma. In fact, nothing so drastic is necessary. For example, it was appreciated early by several people that matter under the extreme physical conditions holding sway at early cosmological times may, already at the classical level, violate the strong energy condition, and thus help to sidestep the singularity theorems.

In this spirit cosmological models have been exhibited in which the initial singularity is absent thanks to the matter in the universe exhibiting at high densities either large viscosity (Murphy, 1973) or negative pressure (Rosen, 1974). Today most would agree that postulating special properties of matter to exorcise the singularity is an approach with limited credibility attaching to it. However, the fundamental point can be made in a more basic way.

Interactions between particles in the early universe must be important. The nuclear interactions may be regarded as mediated by the Yukawa field, a massive scalar field. Such a field can trivially violate the energy condition (1) already at the level of classical physics (work it out!). Thus, one suspects that the condition will be violated at early times when the matter is at nuclear or higher density ($z > 10^{14}$). One might try to model the relevant universe by including a homogeneous Yukawa field in addition to the usual matter and radiation.

Since it is mathematically complex to solve the Friedmann equations in the presence of a massive scalar field, I once looked (Bekenstein, 1975) for exact cosmological solutions to Einstein's equations representing universes filled with pressureless matter, radiation, and a homogeneous *conformal* scalar field minimally coupled to the matter. The conformal scalar field also violates condition (1), so it can be argued that this is a suggestive model for the real situation. I found models which bounce away from the singularity for all three space topologies. This all depends on the relation between the three parameters describing the strength of the coupling, and the amounts of matter and radiation present, and three integration constants. Particularly aesthetic models can be had for the positive-curvature universe; in these the conformal field's sole source is the matter in the model (and not an initial condition), in agreement with the motivating picture of this approach.

Models in the same spirit, and taking account of the strong interaction in more realistic fashion, are also known (Dehnen and Honl, 1975). All the above examples show that the singularity may be removable at the level of classical physics by the natural properties of the matter contained in the universe. It must, however, be emphasized that not every model of this kind is nonsingular.

4. DEPARTURES FROM CLASSICAL GENERAL RELATIVITY

The second road toward excising the cosmological singularity seeks to modify general relativity (GR) itself. Much effort has gone into generalizing the Hilbert action of GR to include quadratic terms in the curvature (Ginzburg, 1971; Nariai and Tomita, 1971). Nariai and Tomita have exhibited explicit nonsingular models based on such a gravitational theory.

It is probably fair to say that most theorists would reject such a modification of Einstein's equations if it is *ad hoc* and made for the sole purpose of eliminating the singularity. Of course, we now know that terms quadratic in the curvature can appear in the action in the wake of regularization of quantum field divergences. However, that is another story to be mentioned a bit later. The point I want to make here is that some modifications of GR can come from a separate and more reasonable motivation.

Let me illustrate by briefly describing the variable mass theory (VMT), a gravitational theory I proposed (Bekenstein, 1977) to assess the extent to which the predictions of GR are dictated by the strong equivalence principle. Among other things, this principle would require the rest masses of various particles to be constant in Planck-Wheeler units, i.e., the ratio of a rest mass to $(\hbar c/G)^{1/2}$ must be a spacetime constant. VMT dispenses with this particular *assumption*, and allows masses to have dynamics of their own. Under very general conditions it can be shown that rest masses must vary as a real power of a scalar field which may have a coupling to curvature, and whose source is the matter's T. (In this sense VMT is the ultimate extension of Brans and Dicke's theory; the general scalar tensor theories drift away from the original intent). Despite its more liberal axioms and different structure, VMT has the peculiarity of mimicking many of the predictions of GR for the solar system tests of relativity (Bekenstein, 1977), and for neutron stars and black holes (Bekenstein and Meisels, 1978). And, unlike Brans-Dicke theory, it does this without the benefit of a parameter which must be adjusted to have large values.

Insofar as observable predictions are concerned, VMT and GR are pretty much indistinguishable experimentally with present technology. Indeed, the paper which described the similarity (Bekenstein and Meisels, 1978) bore the title "General relativity without general relativity." Anyway, unlike the standard Friedmann models, many VMT cosmological models which tend to behave like the Friedmann models at late epochs (Bekenstein and Meisels, 1978) are nonsingular (Bekenstein and Meisels, 1980). Thus, one has the best of possible worlds: GR behavior almost across the board, with the option of a nonsingular universe. Again, I have to emphasize that there are also singular VMT models. The singularity is not compulsory, but neither is its elimination.

5. QUANTUM FIELD EFFECTS

Most relativists do not like a classical modification of GR, even one as conservative as VMT. The majority view has for long sought to put the blame for the appearance of the singularity on the unjudicious application of classical physics in the very early stages of the universe. It seems to have been recognized early in the 1970s that quantum field-theoretic effects can violate the strong energy condition. (For comparison recall that effects like these violate the related weak energy condition and thus allow a black hole to circumvent the Hawking area theorem and emit Hawking radiation). With the fall of the condition, the singularity theorems lose their force and this opens a third road toward elimination of the initial singularity.

Thus, it was shown (Fulling and Parker, 1973) that if the universe contains a massive scalar field prepared in a coherent quantum state, the singularity is avoidable by a cosmic bounce. Of course, one is left with the problem of what mechanism will favor such quantum states. Recall also that the massive scalar field already violates the energy condition classically, so presumably a bounce could also be obtained with no appeal to quantum effects. The bounce solution to the singularity problem is also implemented in several models in which the trace anomaly of quantum scalar fields plays a crucial role (Fischetti *et al.*, 1979; Anderson, 1983). The universe can bounce because the regularized stress-energy tensor of the fields does not respect the strong energy condition. However, not every possible model obtained by this strategy is nonsingular.

It is possible to escape the singularity with no recourse to a cosmic bounce. In one charming model universe (Starobinsky, 1980), the expansion is driven by vacuum polarization of various conformal fields which are all in the vacuum state. In its initial nonsingular state this universe is thus empty. It expands in accordance with de Sitter's solution, i.e., undergoes inflation, before embarking on a standard Friedmann-like expansion when particles finally appear in it. The de Sitter phase is nonsingular and so the Starobinsky model avoids the singularity. Another model universe (Vilenkin, 1983) materializes out of "nothing" with already finite curvature by virtue of instanton effects of the Higgs field it contains. It thus avoids the singularity. This model is still not quantum cosmology in the literal sense, for the gravitational field is treated as classical; for a quantum gravity version see Vilenkin (1989). All the above investigations and those of several other researchers show that, in the presence of quantum field effects, the cosmological singularity is no longer compulsory. But, as we have seen, the generic model with quantum fields is not guaranteed to be nonsingular.

6. QUANTUM COSMOLOGY

The fourth road toward exorcising the cosmological singularity is quantum gravity. It is no exageration to say that most gravity theorists expect this to be the final answer to the singularity problem. And yet no finished theory of quantum gravity exists. At the present stage various elements and tools of this theory have taken shape without it being clear how they will all fit together into a coherent apparatus. Quantum gravity in its final form should deal with the probability amplitudes (wavefunctions) for this or that spatial geometry to appear. Given the appearance of the Planck-Wheeler (PW) scale in the theory, it is expected that spacetimes in which the curvature can become larger than the inverse squared PW length, or can vary rapidly on that scale, should become highly improbable. This would be the quantum gravity version of a prohibition on singularities such as the Friedmann singularity.

Does the nascent quantum gravity framework paint a picture of the eradication of the cosmological singularity in consonance with the expectations? Over the last two decades there have been several approaches to quantum gravity, and numerous attempts (too many to review here if only in a qualitative way) to build nonsingular cosmological models in this way. Here I mention only the approach based on "functional integral" quantization pioneered by Hawking and his school (Hawking, 1979; Hawking and Halliwell, 1985; Hartle, 1983). Cosmological models built on this basis sometimes give the hoped-for vanishing of the probability amplitude at a curvature singularity. But far from being generic, this result is contingent on a particular choice of factor ordering in the Wheeler–DeWitt equation, the quantum gravity analog of the Schrödinger equation. There are possible (even reasonable) factor orderings for which the amplitude at a singular surface is nonvanishing (Hartle, 1983).

A simpler version of the functional quantization approach that quantizes only the conformal factor in the metric (Narlikar and Padmanabhan, 1986) also leads to vanishing probability amplitude for a singularity for "most" models, but one is left to wonder whether this is not an artifact of the restriction in the number of dynamical degrees of freedom. Thus, in common with the other roads, the quantum gravity road has not yet led to an unambiguous destination.

7. THE BOUND ON ENTROPY

The situation is thus a frustrating one. It would be nice to know what the verdict on the singularity is without waiting many more years for the full-blown formalism of quantum gravity to crystallize. Fortunately, there is perhaps a way! Historically, whenever consideration of detailed physical mechanisms has failed to render verdict on the reality of some disputed phenomenon, physicists have often resorted to thermodynamics to settle matters. As is well known, thermodynamics gives relations between macroscopic quantities whose validity is independent of the form of the microphysics on which they ultimately depend. Thus, the temptation is great to bring thermodynamics to bear on the question of the cosmological singularity.

Of course, not every physical issue can be handled by thermodynamics. Might not that be the case in our problem? Indeed, the view has usually been that thermodynamics cannot be invoked to cast doubt on the reality of the singularity because there is nothing contradictory about employing thermodynamics to describe the contents of a cosmological model which extrapolates to a singularity in the past. This view is already present in Tolman's classic treatise (Tolman, 1934), and is futher elaborated in modern texts (Weinberg, 1972). However, I believe that it oversimplifies the issue, and that thermodynamics actually supports the view that there was never a singular event in the universe's history. Missing in the usual thermodynamic treatments of the expanding universe is a supplement to the second law of thermodynamics of rather recent vintage.

The second law states that the entropy of a closed system cannot decrease, and indeed tends to a maximum. In its traditional form the law is silent about the numerical value of the maximum. I have suggested (Bekenstein, 1981*a*) that if a complete physical system can be enclosed in a sphere of radius R, then $2\pi R/\hbar c$ sets an upper bound on the ratio of the maximum entropy S that the system may contain to its total energy E (I regard entropy as dimensionless; hence here Boltzmann's $k=1$). This bound supplements the second law in that it constrains the maximum value of the entropy, albeit not very onerously. Indeed, for "household" thermodynamic systems the bound is true almost trivially, and is unenlightening. However, for systems in which quantum discreteness asserts itself, the bound can make nontrivial predictions.

The existence of this bound was originally suggested by application of the generalized second law of thermodynamics (Bekenstein, 1973; Hawking 1975) to *gedanken* experiments involving thermodynamic systems with a black hole (Bekenstein, 1981*a*). It was later pointed out (Unruh and Wald, 1982) that such arguments cannot constitute a general proof of the bound. No such proof was forthcoming in the early days of the subject; for years one's faith in the bound was based on a variety of examples supporting it, particularly for systems composed of free quantum fields (Bekenstein, 1983, 1984), or many quantum mechanical particles (Qadir, 1983; Kahn and Qadir, 1984). Successes of the bound in systems with strong gravitational fields (Sorkin *et al.*, 1981), with quartic field interaction (Bekenstein and Guendelman, 1987), and superstring systems (Bowick *et al.*, 1986) were also known. Recently a deductive proof of the bound for free quantum fields enclosed in an arbitrary cavity has been given (Schiffer and Bekenstein, 1988). That proof may be extendable to a fairly broad class of interacting fields (Schiffer, 1988).

In its early days the bound was used to predict a limit on the number of generations of quarks (Bekenstein, 1982), and a bound on the rate of communication possible within a given energy budget through a channel (Bekenstein, 1981*b*, 1982). Today the evidence from cosmology indicates that the number of generations is probably limited to four, while detailed work (Bekenstein, 1988) has confirmed a facet of the predicted bound on communication rate. In view of these successes of the bound, and of the newer proofs of its validity, it seems timely to apply the bound to the issue of the cosmological singularity. As will be clear below, the bound predicts that there was no singularity of the Friedmann type.

To show this I shall work with a radiation-dominated Friedmann model, and show that a contradiction with the bound is imminent if the model is extrapolated too far back in time. There is even a suggestion that quantum gravity is the physics that prevents the singularity. However, nothing in the way of a minimal scale below which spacetime cannot be regarded as smooth will be assumed in the argument. Indeed, in view of the point of view introduced by superstring theory, it is better not to rely too heavily on this assumption.

First, some problems must be overcome. The bound is meant to apply to a system of finite size; the universe, for all we know, may be infinite. This problem would be resolved by applying the bound to an arbitrary finite region of the universe, but of course such region could not, in general, be regarded as a complete system, as required by the bound (Bekenstein, 1981*a*). In addition, the bound has been tested mostly for static systems, but the universe expands. The energy of the system is a key ingredient of the bound, but the energy of the full universe may not be a meaningful concept. Finally, apart from one example (Sorkin *et al.*, 1981), very little is known about the validity of the bound in the presence of spacetime curvature.

8. ROLE OF THE PARTICLE HORIZON

I propose to sidestep some of these problems by applying the entropy bound, not to a comoving region or to the whole universe, but to a region inside the particle horizon of a definite observer. As is well known (Narlikar and Padmanabhan, 1986), all observers in a Friedmann model have their vision limited by the finiteness of the speed of light, and by the fact that no propagation of information preceded the cosmological singularity.

To make this precise, consider the metric in a radiation-dominated Friedmann model at early times:

$$ds^2 = -c^2\,dt^2 + b^2 t[dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)] \tag{2}$$

Here b is a constant with no direct physical significance. The singularity is at $t=0$ for all space points. For a given observer O situated at $r=0$ at

$t = t_0$, the backward light cone is delineated by integrating the condition $ds^2 = 0$. It is the surface

$$t = (\sqrt{t_0} - br/2c)^2 \tag{3}$$

labeled H in Figure 1. Along this surface t decreases with increasing r, eventually vanishing at $r = 2c\sqrt{t_0}/b$. This is how far from $r = 0$ a signal may start at $t = 0$ and hope to reach O at $t = t_0$. Obviously, O cannot, at time t_0, see events that occurred at $r > 2c\sqrt{t_0}/b$. The hypersurface (3) which encloses the part of the universe visible to O is therefore called the *particle horizon* of O.

Ordinarily the entropy bound is applied to a system enclosed by some material boundary. Here I shall instead apply it to the material content (radiation in our specific example) of the section of some spacelike hypersurface which lies entirely within the particle horizon of O, e.g., hypersurfaces Σ and Σ' in Figure 1. Such a spacelike region has been influenced only by that part of the early universe which has also affected O. Hence, with reference to our particular observer O, it is the next best thing to a closed system. In this way we dispose of two of our problems: we have a "closed" system of finite size.

Another of our problems relates to the lack of steady state. I think this is not problematic in our situation. We may assume for the stages just after

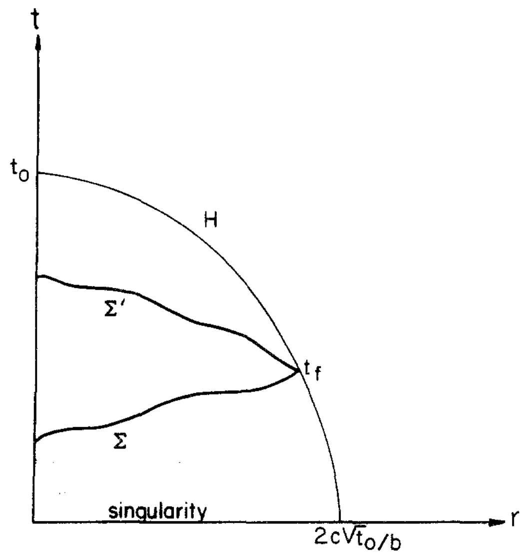

Fig. 1. The particle horizon H of observer O living at time t_0, and sections of two spacelike hypersurfaces Σ and Σ' entirely within H and intersecting it at time t_f.

the singularity, when all processes proceed in thermodynamic equilibrium, that entropy is conserved. (Gravitons should soon fall out of equilibrium with the other contents of the universe; however, since they are massless, their entropy is preserved during the expansion.) This means that

$$S^{\mu}{}_{,\mu} = 0 \tag{4}$$

where S^{μ} is the entropy current density. It is then clear that the entropy S contained in Σ, namely

$$S(\Sigma) = \int_{\Sigma} S^{\mu}\, d\Sigma_{\mu} \tag{5}$$

(here $d\Sigma_{\mu}$ is the three-dimensional hypersurface element erected on Σ) is unaffected if Σ is defined in any way that does not displace its junction point with H, e.g., replacing Σ by Σ' in Figure 1. The fact that space is expanding thus loses much of its significance: insofar as the entropy is concerned, we can work with a hypersurface which samples the spacetime at various times t.

Another problem in our list is the definition of the energy of the system. Energy is not a scalar, so we need definite observers to specify it. It seems inappropriate to try to deal with gravitational energy in part of a homogeneous space, so I will assume that energy in the bound is to be interpreted as thermal energy. Then the following definition of energy E seems appropriate. If u^{μ} is the four-velocity field of the radiation in question [in the coordinates used in (2), $u^{\mu} = (c^{-1}, 0, 0, 0)$], we first construct the Poynting vector $-T^{\nu}_{\mu} u^{\mu}$ out of the stress-energy tensor for the radiation

$$T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} + p g^{\mu\nu} \tag{6}$$

(the radiation energy density ρ and pressure p being related by $p = \rho/3$) and then write the energy as

$$E(\Sigma) = -\int_{\Sigma} T^{\nu}_{\mu} u^{\mu}\, d\Sigma_{\nu} \tag{7}$$

Of course, $E(\Sigma)$ is not invariant under deformations of Σ. As shown in the Appendix, as a consequence of the assumed positiveness of p, E will decrease if Σ is deformed into the future.

What about the problem posed by the curvature? With regard to *spatial* curvature, there is no problem. As is well known, in Friedmann models at early times space curvature is negligible in the sense that the spatial radius of curvature is very large compared to the size of the horizon. It is this very fact which allows us to represent all three types of spaces (flat, closed, hyperbolic) by one metric, namely (2). *Spacetime* curvature is not covered by this excuse; I will just have to assume that the bound is still valid in its presence.

9. ENERGY AND ENTROPY

To calculate the entropy (5) and energy (7) on a section of an hypersurface like Σ in Figure 1, recall that in the Friedmann models $S = su^{\mu}$, where s is the scalar entropy density. According to the Boltzmann formulas, the contribution of *each species* of massless boson quanta with two helicity states is

$$\rho = a_{\mathrm{SB}} T^4 \qquad \text{and} \qquad s = 4/3\; a_{\mathrm{SB}} T^3 \tag{8}$$

where T is the temperature and $a_{\mathrm{SB}} = \pi^2/15\hbar^3 c^3$ is the Stefan–Boltzmann constant (each helicity state of a massless fermion species contributes 7/16 as much). Substituting (8) in Friedmann's equation

$$a^{-2}(da/dt)^2 = (8\pi G/3)\rho c^{-2} \tag{9}$$

we get

$$T = c^{5/4}\hbar^{3/4}(GN)^{-1/4} t^{-1/2} \tag{10}$$

where N is the effective number of particle species present in the early universe (because T is high, we regard all species as massless). In (10) and subsequently *I have left out all factors like* π, $\sqrt{2}$, etc., in the interest of transparency. By substituting (10) in (8), one may evaluate the integrals (5) and (7) once Σ is specified.

As mentioned, S is invariant under deformations of Σ, while E decreases as Σ is deformed into the future. Hence the highest S/E for surfaces Σ having a fixed intersection with H obtains for Σ approaching the particle horizon to the future of the intersection. I shall thus focus on a hypersurface which coincides with H to the future of its footpoint $t = t_f$ (see Figure 1; mathematically it makes little difference that the limiting surface is null). What we shall be doing is computing S and E of all the radiation visible in a snapshot of the whole sky obtained by O at time t_0 which records back to time t_f. This procedure has much in common with the astronomer's interest in all objects visible in a given photographic plate. There, too, only objects on the light cone are seen, and there is a practical cutoff in depth which often coincides, for very "deep" plates, with a redshift or footpoint cutoff.

To evaluate the integrals on the limiting null surface (3), first note that since the vector integrands in (5) and (7) have only a time component, the only relevant component of $d\Sigma_{\mu}$ is

$$d\Sigma_t = cb^3 t^{3/2} r^2 \sin\theta \, dr \, d\theta \, d\varphi \tag{11}$$

in which we must express t in terms of r by means of (3). Carrying out the integrals from $r = 0$ to $r = r_f \equiv (2c/b)(\sqrt{t_0} - \sqrt{t_f})$ [according to (3), this is the comoving radial coordinate corresponding to the footpoint t_f], we get

$$S(\Sigma) = c^{15/4}(\hbar G)^{-3/4} N^{1/4}(\sqrt{t_0} - \sqrt{t_f})^3 \tag{12}$$

$$E(\Sigma) = c^5 G^{-1}[1/2 t_0 \ln(t_0/t_f) + 2(t_0 t_f)^{1/2} - 3/2\, t_0 - 1/2\, t_f] \tag{13}$$

Not unexpectedly, the arbitrary scale factor b has dropped out from the final results.

Consider now the dependence of S/E on t_f for fixed t_0. Numerically, one finds that the ratio grows monotonically from zero at $t_f = 0$ to a finite constant at $t_f = t_0$. Expanding the square bracket in (13) in the small quantity $\sqrt{t_0} - \sqrt{t_f}$ about $t_f = t_0$, we find analytically the peak value attained by S/E:

$$S/E = (G/\hbar^3 c^5)^{1/4} N^{1/4} \sqrt{t_0} \tag{14}$$

This result may be interpreted as the largest S/E observable to O. Although much the same value value would be obtained by just taking s/ρ at fixed time t_0, in that case we would lack a clear observational justification for the procedure in terms of a single observer (it amounts to computing the total energy and entropy as reported by many observers all at the same time).

10. CONCLUSIONS

The bound on specific entropy requires that S/E be no larger than $2\pi R/\hbar c$, where R is the largest radius of the system in question. It seems most appropriate to interpret R as the metric radius R_H at time t_0 of the region between $r = 0$ and $r = 2c\sqrt{t_0}/b$, since this is the region which has causally influenced the radiation whose entropy and energy we have computed. From the metric (2) we find that

$$R_H = 2ct_0 \tag{15}$$

Evidently for t_0 large compared with the Planck-Wheeler time $(G\hbar/c^5)^{1/2}$ the entropy bound is amply satisfied. However, we can also see that *if we put our fiducial observer O too early*, namely at $t < \sqrt{N}(G\hbar/c^5)^{1/2}$, then *the entropy bound is contradicted.*

If we put trust in the entropy bound, then there are several ways out of the dilemma, all of which amount to denying the existence of the cosmological singularity. First, one might argue that there was no particle horizon at all in the universe, thus eliminating the natural basis for application of the bound. For example, if the universe started off expanding in a precisely de Sitter model, as Starobinsky would have it, there is no particle horizon, and neither is there a cosmological singularity as such. Likewise, if the universe entered into expansion after having first contracted and then bounced, as in the VMT cosmological models, or in the Fulling-Parker or Anderson models, then there is no causal barrier, no horizon, and no singularity.

Alternatively, were the universe to materialize out of nothing somewhat after the fashion of Vilenkin's model, we might have a situation where the region visible to an observer is limited in dimension, but is never smaller than some minimum size dictated by the radius of curvature of the emergent spacetime. Here, again, there would be no singularity, but there might be something like a particle horizon. Application of the bound on entropy to the appropriate region might then tell us that its dimension cannot be smaller than the Planck-Wheeler length $(G\hbar h/c^3)^{1/2}$, and indeed may be required to be large compared to it if N, the number of species in the early universe, is itself large.

Thus, whichever alternative is relevant, thermodynamics leads to a conclusion in harmony with expectations based on our vision of full-fledged quantum gravity: it makes no sense to think of spacetime with an arbitrarily high curvature event. The appearance of the Planck-Wheeler quantities in determining the earliest time for an observer suggests that it is quantum gravity which is responsible for the prevention of singularity. However, as stressed earlier, thermodynamic conclusions are not dependent on detailed mechanisms. We cannot be sure that the present universe was coaxed away from a singularity by quantum effects in the gravitation, rather than by one of the other mechanisms reviewed earlier. What we do get out of thermodynamics is an assurance that the standard Friedmann models, so useful for describing the recent universe, cannot be extrapolated all the way back to their singular beginnings. This is found to be inconsistent with thermodynamics.

APPENDIX

Here we obtain the change of the energy E defined by (7) when the hypersurface Σ is deformed into Σ' (see Figure 1). In view of the divergenceless nature of $T^{\mu\nu}$, we can write by means of Gauss' theorem

$$\Delta E \equiv E(\Sigma') - E(\Sigma) = -\int_V T^{\mu\nu} u_{\nu;\mu} (-g)^{1/2}\, d^4x \tag{A1}$$

where V is the 4-volume enclosed between Σ and Σ'. The contribution from the surface at $r=0$ in Figure 1 vanishes because of the factor r^2 in the 3-volume element. From Raychadhury's theorem we have

$$u_{\nu;\mu} = -a_\nu u_\mu + \sigma_{\nu\mu} + \omega_{\nu\mu} + \tfrac{1}{3}\theta(g_{\nu\mu} + u_\nu u_\mu) \tag{A2}$$

where a_ν is the acceleration of the flow field u_ν, $\sigma_{\nu\mu}$ is its the shear tensor, $\omega_{\nu\mu}$ its rotation tensor, and $\theta \equiv u^\mu{}_{;\mu}$ its expansion scalar. Because a_ν and u_μ are orthogonal and $T^{\mu\nu}$ has the form (6), the a_ν term of (17) does not contribute to the integrand of (16). Likewise, the rotation term drops out by virtue of the symmetry of $T^{\mu\nu}$ and the antisymmetry of $\omega_{\nu\mu}$. The shear term also drops out because $\sigma_{\nu\mu}$ is purely spacelike and traceless. The remaining term gives

$$\Delta E = -\int_V \theta p(-g)^{1/2}\, d^4x \tag{A3}$$

from which it is clear that since $\theta > 0$ in a Friedmann model, and the pressure is assumed positive, then E on Σ' is smaller than on Σ if Σ' is completely to the future of Σ.

NOTE ADDED IN PROOF

In a recent Sao Paolo preprint M. Schiffer has extended the argument given here to anisotrophic cosmological models.

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